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Your Roll No.....

Sr. No. of Question Paper : 4997

E

Unique Paper Code : 62351201

Name of the Paper : Algebra

Name of the Course : B.A. (Prog.)

Semester : II

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions.
3. Attempt any two parts from each question.
4. Marks are indicated against each question.

1. (a) Form an equation whose roots are $-1, 2, 3 \pm 2i$. (6)

(b) Solve the equation (6)

$$x^3 - 13x^2 + 15x + 189 = 0,$$

being given that one of the roots exceeds another by 2.

(c) If α, β, γ , be the roots of the equation (6)

$$x^3 + 5x^2 - 6x + 3 = 0, \text{ find the value of}$$

P.T.O.

$$(i) \sum \alpha^3$$

$$(ii) \sum (\alpha - \beta)^2.$$

2. (a) Prove that :

(6.5)

$$2^{10} \cos^6 \theta \sin^5 \theta = \sin 11\theta + \sin 9\theta - 5 \sin 7\theta - 5 \sin 5\theta + 10 \sin 3\theta + 10 \sin \theta.$$

(b) Sum the series :

(6.5)

$$\cos \alpha + \cos (\alpha + \beta) + \cos (\alpha + 2\beta) + \dots \text{ to } n \text{ terms, provided } \beta \neq 2k\pi.$$

(c) State DeMoivre's theorem for rational indices and use it to solve the equation :

(6.5)

$$x^7 - x^4 + x^3 - 1 = 0.$$

3. (a) Find the characteristic roots of the matrix A where

(6)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$

(b) Solve the system of linear equations

(6)

$$2x - 5y + 7z = 6$$

$$x - 3y + 4z = 3$$

$$3x - 8y + 11z = 11$$

- (c) Using Cayley Hamilton's Theorem, find the value of A^3 , where (6)

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$$

4. (a) Let X and Y be two subspace of a vector space V . (6.5)

(i) Prove that the intersection $X \cap Y$ is also subspace of V .

(ii) Show that the union $X \cup Y$ need not be a subspace of V .

- (b) Let $V = F[a, b]$ be the set of all real valued functions defined on the interval $[a, b]$. For any f and g in V , c in R , we define

$$(f + g)(x) = f(x) + g(x),$$

$$(c.f)(x) = cf(x)$$

Prove that V is a vector space over R , where R denotes the set of real numbers. (6.5)

- (c) Show that the vectors $v_1 = (1,1,1)$, $v_2 = (1,1,0)$, $v_3 = (1,0,0)$ form a spanning set of $R^3(R)$, where R denotes the set of real numbers. (6.5)

5. (a) Find the multiplicative inverse of the given elements (if it exists) if it does not exist, give the reason

(i) $[12]$ in Z_{16} (ii) $[38]$ in Z_{83} (6)

- (b) Find the order of each of the following permutations

(i) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 5 & 3 & 2 & 1 \end{pmatrix}$

(ii) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 7 & 5 & 1 & 8 & 2 & 3 \end{pmatrix}$

- (c) Let G be a group. Prove that G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$. (6)

6. (a) Prove that the set $S = \{0, 2, 4, 6, 8\}$ is an abelian group with respect to addition modulo 10. (6.5)

- (b) Let G be the group of all 2×2 invertible matrices with real entries under the usual matrix multiplication. Show that subset S of G defined by

$$S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, b=c \right\}, \text{ does not form a subgroup of } G. \quad (6.5)$$

- (c) Show that $Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$ is a subring of R , where R is a set of real numbers & Q is set of rational numbers. (6.5)