[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 831 G

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : B.Sc. (Physical Science and Mathematical Science) with Operational Research and Bachelor of Arts

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting two parts form each question.

3. All questions carry equal marks.

1. (a) Show that the homogeneous equation

$$(Ax^2 + Bxy + Cy^2)dx + (Dx^2 + Exy + Fy^2) = 0$$

is exact if and only if B = 2D and E = 2C. Also solve the initial value problem

$$(2x^2y^2 - y^3 + 2x)dx + (2x^3y - 3xy^2 + 1)dy = 0,$$

$y(-2) = 1$. (7½)

(b) Show that $y = 4e^{2x} + 2e^{-3x}$ is a solution of the initial value problem

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0 \quad y(0) = 6, \quad y'(0) = 2.$$

Is $y = 2e^{2x} + 4e^{-3x}$ also a solution of this problem? Explain why or why not. (7½)

(c) Consider the equation $a\left(\frac{dy}{dx}\right) + by = ke^{-\lambda x}$, where a, b and k are positive constants and λ is nonnegative constant.

(i) Solve this equation.

(ii) Show that if $\lambda = 0$ every solution approaches $\frac{k}{b}$ as $x \to \infty$, but if $\lambda > 0$ every solution approaches 0 as $x \to \infty$.

(7½)

2. (a) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = 100$ and family of Parabolas $y = 10x^2$. (7½)

(b) The population x of a certain city satisfies the logistic law

$$\frac{dx}{dt} = \frac{1}{100x} - \frac{1}{(10)^8}x^2$$

where time t is measured in years. Given that the population of this city is 100,000 in 1980, determine the population as a function of time for $t > 1980$. In particular answer the following questions :

(i) What will be the population in 2000?

(ii) In what year the does the 1980 population double? (7½)

(c) Solve the Homogeneous differential equation

$2r(s^2 + 1)dr + (r^4 + 1)ds = 0.$ (7½)

3. (a) Find the general solution of

$$\left(x^2 + 2x\right)\frac{d^2y}{dx^2} - \frac{2(x+1)dy}{dx} + 2y = 1$$

Given that $y = (x + 1)$ and $y = (x + 1)^2$ are linearly independent solutions of the corresponding homogeneous equation. (7½)

(b) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+4y=2x\ln x, \qquad x>0 \qquad (7\tfrac{1}{2})$$

(c) Define Quasi–linear, Semi–linear and linear first order partial differential equation and give one example each. Also show that a family of spheres $x^2 + y^2 + (z-c)^2 = r^2$ satisfies the first order linear partial differential equation $y\frac{dz}{dx}-x\frac{dz}{dy}=0$.

(7½)

4. (a) Use the method of variation of parameter to find a particular solution of the differential equation :

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=\frac{e^{-3x}}{x^3}. \qquad (7\tfrac{1}{2})$$

(b) Use the method of undetermined coefficients to find the particular solution of the differential equation :

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}-4y=16x-12e^{2x}. \qquad (7\frac{1}{2})$$

(c) Given that $y = e^{2x}$ is a solution of

$$(2x+1)\frac{d^2y}{dx^2}+4(x+1)\frac{dy}{dx}+4y=0.$$

Find a linearly independent solution by reducing the order. Write the general solution. (7½)

5. (a) Find the solution of Cauchy problem for first order PDE.

$$2\frac{\partial u}{\partial x}+3\frac{\partial u}{\partial y}+z=0 \text{ with } u(x, 0) = \sin x \qquad (7\frac{1}{2})$$

(b) Find the Solution of characteristic equation for the first order PDE.

$$\frac{\partial u}{\partial x}+2x\frac{\partial u}{\partial y}=2xu \text{ with } u(x, 0) = x^2 \qquad (7½)$$

(c) Find the general solution of the equation :

$$(cy - bz)zx + (az - cz)zy = bx - ay \qquad (7½)$$

6. (a) Solution general solution of Cauchy problem for first order PDE.

$$\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial y}=\left(\frac{y-1}{2x^2}\right)^2 \text{ with } u(0, y) = \exp(y) \qquad (7½)$$

(b) Find the general solution of the partial differential equation :

$$x^2(y - u)u_x + y^2(u - x)u_y = u^2(x - y) \qquad (7½)$$

(c) Reduce the equation :

$y^2 u_{xx} + x^2 u_{yy} = 0$, $x \neq 0$, $y \neq 0$, find the general solution. (7½)