

Your Roll No.....

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Unique Paper Code : 2352203502

Name of the Paper : DSC-5 : LINEAR PROGRAMMING

Name of the Course : **B.A. (Prog.) Mathematics**
with Major

Semester : V

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Parts of questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator is not allowed.

P.T.O.

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1. (a) (i) A person wants to decide the constituents of a diet which will fulfill his daily requirements of proteins, fats and carbohydrates at minimum cost. The choice is to be made from four different types of foods. The yield per unit of these foods are given in the table below.

Food Type	Yield/unit			Cost/Unit Rs
	Proteins	Fats	Carbohydrates	
1	3	2	6	40
2	4	2	4	40
3	8	7	7	80
4	6	5	4	60
Minimum Requirement	800	200	700	

Formulate the LP for the problem to find the food type mix to minimize the cost.

- (ii) Test the convexity for the set $S = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \leq 1, x_2 \leq 4\}$

- (b) Find all the basic feasible solutions of the following system of equations :

$$2x_1 + x_2 - x_3 + 3x_4 = 2$$

$$x_1 + 4x_2 + x_3 + x_4 = 1$$

- (c) Solve the following LPP graphically

$$\text{Maximize } z = 5x_1 + 4x_2$$

Subject to

$$x_1 + 2x_2 \leq 6$$

$$-2x_1 + x_2 \leq 4$$

$$5x_1 + 3x_2 \leq 15$$

$$x_1, x_2 \geq 0$$

2. (a) Solve the following LPP using simplex method

$$\text{Maximize } 2x_2 + x_3$$

$$\text{subject to } x_1 + x_2 - 2x_3 \leq 7$$

$$-3x_1 + 2x_2 + 2x_3 \leq 3$$

$$x_1, x_2, x_3 \geq 0$$

(b) Solve the following LPP using simplex method

$$\text{Minimize } -4x_1 + x_2 + x_3 + 7x_4 + 3x_5$$

$$\text{Subject to } -6x_1 + x_3 - 2x_4 + 2x_5 = 6$$

$$3x_1 + x_2 - x_3 + 8x_4 + x_5 = 9$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

(c) Solve the following LPP using simplex method

$$\text{Maximize } 3x_1 - x_2$$

$$\text{subject to } x_1 - x_2 \leq 3$$

$$2x_1 - x_2 \leq 0$$

$$x_1 + x_2 \geq 12$$

$$x_1, x_2 \geq 0$$

3. (a) Solve the following LPP using Simplex method

$$\text{Maximize } x_1 - 2x_2$$

$$\text{subject to } x_1 + x_2 \geq 2$$

$$-x_1 + x_2 \geq 1$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

- (b) Write the dual of the following primal problem

$$\text{Minimize } z = 3x_1 + x_2 + x_3 - 2x_4$$

$$\text{Subject to } x_1 - 2x_2 + 4x_3 - 2x_4 = 2$$

$$2x_1 - 2x_2 + x_4 \leq 3$$

$$2x_2 - x_3 + x_4 \geq 5$$

$$x_1, x_2, x_3, x_4 \geq 0$$

- (c) Find the dual of the following LPP:

$$\text{Minimize } 8x_1 + 3x_2 - 2x_3$$

$$\text{subject to } 3x_1 - 6x_2 + x_3 = 2$$

$$3x_1 + 7x_2 - 2x_3 \geq 4$$

$$x_1, x_2 \geq 0, x_3 \text{ unrestricted in sign}$$

4. (a) Find the optimal transportation routes and minimum total transport cost. The following table provides all the necessary information on the availability of supply to each warehouse, the requirement of each

market, and the unit transformations cost (in Rs.) from each warehouse to each market.

	P	Q	R	S	Supply
A	6	3	5	4	22
B	5	9	2	7	15
C	5	7	8	6	8
Demand	7	12	17	9	

- (b) A national truck retail service has surplus of one truck in each of cities 1, 2, 3, 4, 5 and 6; and a deficit of one truck in each of the cities A, B, C, D, E and F. The distance (in Km) between the cities a surplus and cities with deficit are displayed in the table below.

	A	B	C	D	E	F
1	31	62	29	42	15	41
2	12	19	39	55	71	40
3	17	29	50	41	22	22
4	35	40	38	42	27	33
5	19	30	29	16	20	23
6	72	30	30	50	41	20

Determine the optimum assignment schedule.

- (c) Find initial basic feasible solution by using North-West corner rule, Least Cost method and Vogel's approximation method. Compare the solutions.

	A	B	C	Supply
I	2	4	1	40
II	6	3	2	50
III	4	5	6	20
IV	3	2	1	30
V	5	2	5	10
Demand	50	60	40	

5. (a) Certain equipment needs 5 repair jobs which have to be assigned to 5 machines. The estimated time (in hours) that a mechanic requires to complete the repair job is given in the table. Assuming that each mechanic can be assigned only one job, determine the minimum time assignment.

	J1	J2	J3	J4	J5
M1	7	5	9	8	11
M2	9	12	7	11	10
M3	8	5	4	6	9
M4	7	3	6	9	5
M5	4	6	7	5	11

- (b) Starting with Initial Basic feasible solution obtained using Least Cost Method, solve the following transportation problem

Sources	Destinations					Supply
	D1	D2	D3	D4	D5	
S1	48	3	5	2	2	15
S2	6	7	5	5	4	15
S3	5	5	7	3	4	10
Demand	8	4	6	10	12	

- (c) Find saddle points(s), if any, for the following game with payoff matrix :

$$\begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix}$$

Hence, or otherwise solve the game.

6. (a) Consider a game with following payoff matrix

	Player P2			
Player P1	2	6	1	2
	3	5	4	3
	1	0	2	4

Determine the saddle point(s), the optimal strategies for each player, and the value of the game.

- (b) Solve the following using Principle of Dominance

	Player P2		
Player P1	1	2	4
	1	0	5
	0	1	-1

- (c) Transform the following game theory problem into its equivalent pair of linear programming problems for players P1 and players P2:

	Player P2		
Player P1	1	5	-2
	4	1	-3
	2	-1	2