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S. No. of Question Paper : 2564

Unique Paper Code : 2352571101

Name of the Paper : DSC : Topics in Calculus

Name of the Course : B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor

Semester : I

Duration : 3 Hours Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

1. (*a*) Show that the function $f(x) = \begin{cases} x\sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous but not differentiable at $x = 0$.

(*b*) Find the nth differential coefficients of $\sin 3x \sin 2x$.

(*c*) If $u = x^3 + y^3 + z^3 + 3xyz$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 3u$.

2. (*a*) Test the continuity of a function at $x = 0$ which is defined as follows :

$$f(x) = \begin{cases} \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

P.T.O.

(b) If $y = e^{a \sin^{-1} x}\left(\text{or } x = \sin\left[\frac{(\log y)}{a}\right]\right)$,

prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.

(c) If $u = \tan^{-1}\frac{xy}{\sqrt{1+x^2+y^2}}$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{(1+x^2+y^2)^{\frac{3}{2}}}$.

3. (a) State Taylor's theorem with Lagrange's form of remainder. Find the Taylor series expansion of $f(x) = \sin x$.

(b) Evaluate : $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$.

(c) Discuss the applicability of Rolle's theorem for $f(x) = \tan x$ in $[0, \pi]$.

4. (a) If in Cauchy's mean value theorem, $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$, then show that c is the geometric mean of a and b.

(b) Verify Rolle's theorem for :

(i) $f(x) = (x^2 + 2x - 3)e^x$ $\quad x \in [-3, 1]$

(ii) $f(x) = 10x - x^2$ $\quad x \in [0, 10]$.

(c) With the help of Maclaurin's theorem give the expansion of $f(x) = \cos x$ in ascending power of x.

5. (a) Find all the asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1.$$

(b) Find the range of values of x in which the curve :

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7.$$

is concave upwards or downwards. Also find its points of inflexion.

(c) Find the reduction formula for

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx.$$

Hence, evaluate $\int_0^{\frac{\pi}{2}} \cos^6 x \, dx$.

(a) Determine the position and nature of the double points on the curve :

$$y^2 = (x - 2)^2 (x - 1).$$

(b) Trace the curve $y = x^3$.

(c) Obtain a reduction formula for $\int \sin^m x \cos^n x \, dx$. Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x \, dx$.