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Roll No.

S. No. of Question Paper : 2544

Unique Paper Code : 2352571101

Name of the Paper : DSC : Topics in Calculus

Name of the Course : B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor

Semester : I

Duration : 3 Hours Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

1. (*a*) Test the continuity and differentiability of the function $f(x) = \begin{cases} 1+x & \text{if} \quad x \le 2 \\ 5-x & \text{if} \quad x \ge 2 \end{cases}$ at $x = 2$.

(*b*) Find the *n*th differential coefficients of $\sin^3 x$.

(*c*) State Euler's theorem and verify it for $f(x, y, z) = 3x^2yz + 5xy^2z + 4z^4$.

2. (*a*) Show that the function $f(x)$ defined by :

$$f(x) = \begin{cases} 0, & \text{for} \quad x = 0 \\ \frac{1}{2} - x, & \text{for} \quad 0 < x < \frac{1}{2} \\ \frac{1}{2}, & \text{for} \quad x = \frac{1}{2} \\ \frac{3}{2} - x, & \text{for} \quad \frac{1}{2} < x < 1 \\ 1, & \text{for} \quad x = 1 \end{cases}$$

has three points of discontinuity. Find such points and examine the kind of discontinuities.

(b) If $y = \sin(m \sin^{-1} x)$,

show that $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 - m^2) y_n = 0$.

(c) If $u = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}$, prove that $\frac{\partial^2 u}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$.

3. (a) State Lagrange's mean value theorem and use it to show that :

$$\frac{x}{1+x} < \log(1 + x) < x, \quad \text{for} \quad x > 0.$$

Hence show that $0 < \log(1 + x)^{-1} - x^{-1} < 1$, for $x > 0$.

(b) With the help of Maclaurin's theorem give the expansion of $f(x) = \tan x$ in ascending power of x.

(c) Evaluate $\lim_{x \to 0} \frac{x}{|x|}$.

4. (a) Verify Rolle's theorem for the function $f(x) = (x - a)^2 (x - b)^3$ for all $x \in [a, b]$

(b) Separate the intervals in which the polynomial $2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing.

(c) Determine the values of a and b for which

$\lim_{x \to 0} \frac{x(1 + a \cos x) - b \sin x}{x^3}$ exists and equals 1.

5. (a) Find all the asymptotes of the curve :

$$y^3 - x^2 y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0.$$

(f) Find the range of values of x in which the curve :

$$y = 3x^5 - 40x^3 + 3x - 20$$

is concave upwards or downwards. Also find its points of inflexion.

(c) Find the reduction formula for $\int \cos^n x\, dx$.

Hence, evaluate $\int_0^{\frac{\pi}{2}} \cos^7 x\, dx$.

(a) Determine the position and character of the double points on the curve :

$$(x - 2)^2 = y\,(y - 1)^2.$$

(b) Trace the curve $y^2\,(2a - x) = x^3,\ a > 0$.

(c) Obtain a reduction formula for $\int \sin^m x \cos^n x\, dx$. Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^3 x\, dx$.