Name of Course	: **CBCS (LOCF) B.A. (Prog.)**
Unique Paper Code	: **62351101**
Name of Paper	: **Calculus**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find all the points of discontinuity of the function f given by

$$f(x) = \begin{cases} 0 & if\ x = 0 \\ \frac{1}{2} - x & if\ 0 < x < \frac{1}{2} \\ \frac{1}{2} & if\ x = \frac{1}{2} \\ \frac{3}{2} - x & if \frac{1}{2} < x < 1 \\ 1 & if\ x = 1 \end{cases}$$

Also, examine the nature of discontinuity in each case.
Show that the function

$$g(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable when $\sin^{-1} x = 0$.
Find the points at which the function h is not differentiable, where

$$h(x) = |x+1| + 12|2x+5|\ .$$

2. Find the n^{th} differential coefficient of $\tan^{-1} x$.
If $y = e^{a\sin^{-1} x}$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n\ .$$

If $z = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\tan z\ .$$

3. Find the equation of tangents and normal to the curve

$$x = 2a\cos\theta - a\cos 2\theta\ ,\quad y = 2a\sin\theta - a\sin 2\theta \quad at\quad \theta = \frac{\pi}{2}.$$

For the curve

$$y^2(a^2+x^2) = x^2(a^2-x^2)$$

show that origin is a node and that the nodal tangents bisect the angles made by the axes. Also, trace this curve.

4. Find the radius of curvature at the origin for

$$2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0\ .$$

Trace the curve $y = x^3 - (x+1)^2$. Also find the asymptotes of the following curve

$$(x^2 - y^2)^2 - 4x^2 + x = 0\,.$$

5. Show that there is no real number p for which the equation $x^2 - 3x + p = 0$ has two distinct roots in $[0,1]$.

   Show that e^{-x} lies between $1 - x$ and $1 - x + \frac{x^2}{2}$ for all $\in \mathbb{R}$.

   In the Cauchy's Mean Value Theorem on $[a, b](0 < a < b)$

   (i) If $f(x) = \sqrt{x}$ and $g(x) = 1/\sqrt{x}$, then c is the geometric mean between a and b.

   (ii) If $f(x) = 1/x^2$ and $g(x) = 1/x$, then c is the harmonic mean between a and b.

6. Find maxima and minima of the function

$$f(x) = \sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} \quad on \quad [0, \pi]\,.$$

   Show that the maximum value of

$$g(x) = \left(\frac{1}{x}\right)^x \quad is \quad e^{1/e}\,.$$

   Evaluate the function h for maximum and minimum values and separate the intervals of increasing and decreasing, where h is given by

$$h(x) = 2x^5 - 10x^4 + 10x^3 - 15, \qquad x \in \mathbb{R}$$