[This question paper contains 3 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2780 **A**

Unique Paper Code : 62357604

Name of the Paper : Differential Equations

Name of the Course : **B.A. (Prog.) Mathematics : DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1\.

a. Solve : 6

$$(1+yx)xdy+(1-yx)ydx=0$$

b. Solve : 6

$$(y^3-x^2y)dx+\left(3xy^2-\frac{x^3}{3}\right)dy=0$$

c. Solve : 6

$$x+yp^2-p(1+xy)=0 .$$

d. Show that e^{-2x}, e^{-3x} are the linearly independent solutions of 6

$$y''+5y'+6y=0.$$

What is the general solution? Find the solution y(x) with the property $y(0)=0$, $y'(0)=1$.

P.T.O.

2.

a. Solve : 6.5

$$x^3y' - x^2y = -y^4\sin(x).$$

b. Solve : 6.5

$$4y = x^2 + p^2.$$

c. Solve : 6.5

$$xy(y - px) = (x + py)$$

d. Show that e^{-x}, xe^{-x}, e^{2x} are the linearly independent solutions of 6.5

$$y''' - 3y' - 2y = 0.$$

What is the general solution? Find the solution $y(x)$ with the property $y(0) = 0$, $y'(0) = 2$, $y''(0) = 3$.

3.

a. Solve : 6

$$\left(\frac{d^3y}{dx^3}\right) - 3\left(\frac{d^2y}{dx^2}\right) - 6\left(\frac{dy}{dx}\right) = x^2 + 1.$$

b. Solve : 6

$$(x^2D^2 - 3xD + 5)y = x^2 \sin logx .$$

c. Apply the method of variation of parameter to solve : 6

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\, e^x \log x, \qquad x > 0.$$

d. Solve : 6

$$(D^2 + a^2)y = \cos ax.$$

4.

a. Solve the following system of equations : 6.5

$$\frac{dx}{dt} + 2y + x = e^t \text{ and } \frac{dy}{dt} + 2x + y = 3e^t.$$

b. Solve : 6.5

$$\frac{dx}{x^2+2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}.$$

c. Solve : 6.5

$$xz^3dx - zdy + 2y\, dz = 0.$$

d. Solve : 6.5

$$\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}.$$

5.

a. Eliminate the arbitrary function f from the equation: 6

$$z = e^{ax+by} f(ax - by)$$

to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6

$$(x^2 + 2y^2)p - xyq = xz.$$

c. Find the integral surface of the linear partial differential equation 6

$$(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$$

Through the curve $xz=a^3$ and $y=0$.

d. Find the complete integral of the partial differential equation: 6

$$z=px+qy+p^2+q^2.$$

6.

a. (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic: 2.5

$$(x-y)(xr-xs-ys+yt)=(x+y)(p-q),$$

where $r=\frac{\partial^2 z}{\partial x^2}$, $s=\frac{\partial^2 z}{\partial x\partial y}$, $t=\frac{\partial^2 z}{\partial y^2}$, $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$.

(ii) Eliminate the arbitrary constants a and b from the equation : 4

$$2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$

to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6.5

$$px(z-2y^2)=(z-qy)(z-y^2-2x^3).$$

c. Show that the following systems of partial differential equations are compatible and hence solve them 6.5

$$p=x^4-2xy^2+y^4, \qquad q=4xy^3-2x^2y-\sin y.$$

d. Find the complete integral of the partial differential equation: 6.5

$$zpq=p^2(xq+p^2)+q^2(yp+q^2).$$