[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1830 **A**

Unique Paper Code : 32355202

Name of the Paper : GE-2 Linear Algebra

Name of the Course : **Generic Elective**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Do any **two** parts from each question.

Q 1.

(a) If x and y are vectors in $\mathbb{R}^n$, then prove that $\|x+y\| \leq \|x\| + \|y\|$.

Also, verify it for the vectors $x = [-1,3,4,1], y = [3,0,2,-1]$.

(6)

(b) Let x and y be non-zero vectors in $\mathbb{R}^n$, then prove that $\|x+y\| = \|x\|\;\|y\|$ if and only if $y = cx$ for some $c > 0$.

(6)

(c)

(i) Define the projection vector of vector b onto vector a (where a is a non-zero vector).

(ii) For the vectors $a = [4,0,-3]$ and $b = [3,1,-7]$, find $proj_a b$ and verify that $b - proj_a b$ is orthogonal to a.

(6)

(d) Use the Gauss-Jordan Method to find the complete solution for the following system of linear equations

$$-2x + y + 8z = 0$$
$$7x - 2y - 22z = 0$$
$$3x - y - 10z = 0.$$

(6)

Q 2.

(a) Express the vector $[5, 9, 5]$ as a linear combination of the vectors $[2,1,4]$, $[1,-1,3]$ and $[3,2,5]$.

(6.5)

(b) Find the reduced row echelon form matrix B of the following matrix

$$A = \begin{bmatrix} -4 & 2 & 6 \\ 1 & 1 & -3 \\ 2 & 0 & -4 \end{bmatrix}$$

and then give a sequence of row operations that convert B back to A.

(6.5)

(c) Diagonalize the given matrix A, if possible. Here, $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & -5 \end{bmatrix}$.

(6.5)

(d)

(i) Show that the set of vectors of the form $[a, b, 0, c, a - 2b + c]$ in $\mathbb{R}^5$ forms a subspace of $\mathbb{R}^5$ under the usual operations.

(ii) Let V be a vector space, then for any vector $v \in V$ and scalar a, prove that if $av = 0$, then $a = 0$ or $v = 0$.

(6.5)

Q 3.

(a) Find a basis for the row space of the matrix A defined by $A = \begin{bmatrix} 1 & -2 & -3 & 4 \\ 4 & -1 & -5 & 6 \\ 2 & 3 & 1 & -2 \end{bmatrix}$.

(6)

(b) Use the Independence Test Method to show that the subset $S = \{2x^3 - x + 3, 3x^3 + 2x - 2, x^3 - 4x + 8, 4x^3 + 5x - 7\}$ of P_3 is linearly dependent.

(6)

(c) Let $S = \{t + 1, t - 2\}$ and $T = \{t - 5, t - 2\}$ be bases for P_1. If $v \in P_1$, determine $[v]_S$, where $[v]_T = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

(6)

(d) Show that $B = \{[2,3,0,-1], [-1,1,1,-1]\}$ is a maximal linearly independent subset of $S = \{[1,4,1,-2], [-1,1,1,-1], [3,2,-1,0], [2,3,0,-1]\}$. Also calculate dim(span(S)).

(6)

Q 4.

(a) Define a Linear Operator. Show that the projection mapping $L: \mathbb{R}^3 \to \mathbb{R}^3$ given by $L([x, y, z]) = [0, y, z]$ is a linear operator. Also find $L([3,2,-4])$. (6.5)

(b) Find a basis and dimension for the subspace W of $\mathbb{R}^3$ defined by

$$W = \{[x, y, z] \in \mathbb{R}^3 : 2x - 3y + z = 0\}.$$

(6.5)

(c) Let $L: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be a linear transformation and $B = \{[1,1,0], [0,1,0], [-1,1,1]\}$ and $C = \{[-1,1], [1,2]\}$ be bases for $\mathbb{R}^3$ and $\mathbb{R}^2$, respectively. Find the matrix A_{BC} of L w.r.t. B and C.

(6.5)

(d) Use the Simplified Span Method to find a simplified form for span(S) where $S = \{[1,-2,-2], [3,-5,1], [-1,1,-5]\}$.

(6.5)

Q 5.

(a) Consider the linear transformation $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ given by

$L([x, y, z]) = [y, z, -y, 0]$. Show that L is neither one to one nor onto.

(6)

(b) Find the minimum distance from the point P(4,1,-3) to the subspace W=span$\{[x, y, z]: 2x - y + 2z = 0\}$ of $\mathbb{R}^3$.

(6)

(c) Find a least squares solution for the linear system AX = B, where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 5 \\ 1 & 0 & 7 \end{bmatrix}, B = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}.$$

(6)

(d) Consider a polygon associated with 2X 5 matrix $\begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \end{bmatrix}$.

Use ordinary coordinates in $\mathbb{R}^2$ to find the new vertices after performing each indicated operation.

(i) translation along the vector [4,-2].
(ii) rotation about the origin through $\theta = 90^\circ$.
(iii) reflection about the line y= 3x.
(iv) scaling about the origin with scale factors of 4 in the x-direction and 2 in the y-direction.

(6)

Q 6.

(a) Let $L : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ be the linear transformation given by

$$L\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 8 \\ 7 & 1 & 5 \\ -2 & -1 & 0 \\ 3 & -2 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the basis for Ker(L) and a basis for Range(L). Verify Dimension Theorem.

(6.5)

(b) For the subspace W={[$a, b, 0$]: $a, b \in \mathbb{R}$} of $\mathbb{R}^3$, find $W^{\perp}$, the orthogonal complement of W. Verify dim(W)+ dim($W^{\perp}$) =dim($\mathbb{R}^3$).

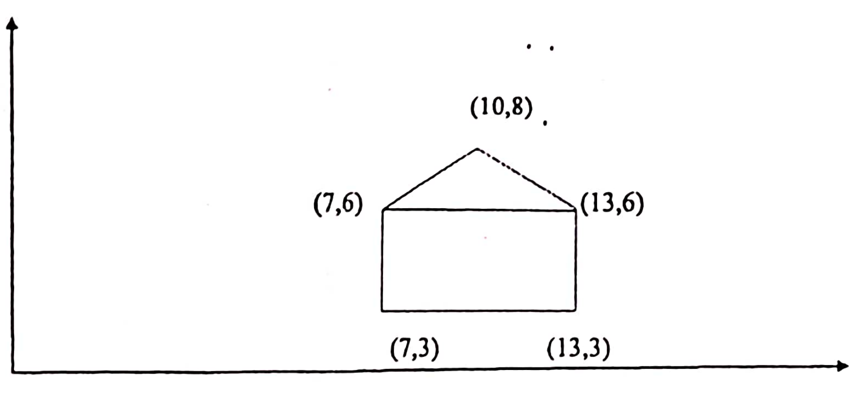
(6.5)

(c) Verify that the given ordered basis is orthonormal.

Hence, for the given v, find $[v]_B$ where v =[-2,1] , B= $\left\{\left[\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right]\left[\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right]\right\}$.

(6.5)

(d) For the given graphic, use homogeneous coordinates to find the new vertices after performing a reflection about the line y = -3x+30. Sketch the final figure resulting from the movement.



(6.5)

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