[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1143 **D**

Unique Paper Code : 2352201102

Name of the Paper : DSC: Elements of Discrete Mathematics

Name of the Course : **B.A. (Prog.)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory. Marks are indicated.

1. (a) Determine the following :

(i) Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$.

(ii) If $p \Rightarrow q$ is false, then determine the truth value of $(\sim (p \wedge q)) \Rightarrow q$. Explain your answer. (7.5)

(b) Let $A = \mathbb{Z}^+$ (the set of positive integers). Define the following relation R on A :

a R b if and only if $|a - b| \leq 2$.

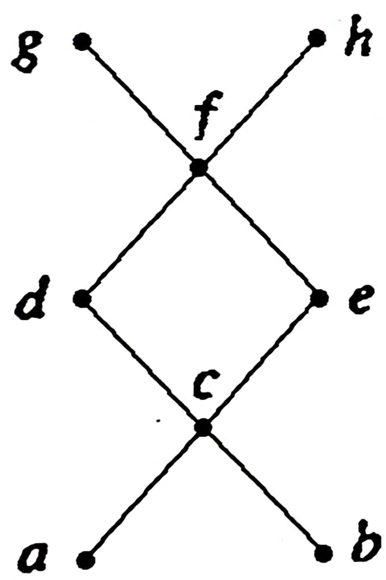
Determine whether the relation R on A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Is R an equivalence relation on A? (7.5)

(c) Prove by mathematical induction that 3 divides $(n^3 - n)$ for every positive integer n. (7.5)

2. (a) For any positive integer n, let D_n denote the set of all positive integers which are divisors of n. Draw the Hasse diagram for D_{12} and D_{15} with the partial order $\leq$ of divisibility defined as $a \leq b$ if and only if a divides b. (7.5)

(b) Consider A = {1,2,3,5,6,10,15,30} and partial order $\leq$ of divisibility on the set A defined as $a \leq b$ if and only if a divides b. Let B = P(S) where S = {e,f,g} be the poset with the partial order ≤' defined as, U ≤' V if and only if $U \subseteq V$ ∀ U, V ∈ B. Show that (A, ≤) and (B, ≤') are isomorphic posets. (7.5)

(c) Find all the maximal and minimal elements, all the lower and upper bounds along with greatest lower and least upper bound of the subset B = {c,d,e} in the following Hasse diagiam. (7.5)



3. (a) Let (L, ∧, ∨) be an algebraic lattice. Define $l \leq m \Leftrightarrow l \wedge m = l$. Show that (L, ≤) is a lattice ordered set. (7.5)

(b) If f is a homomorphism from a lattice L to another lattice M. Show that the homomorphic image of L,f(L) = $\{f(l): l \in L\}$, is a sublattice of M. (7.5)

(c) Define a sublattice of a lattice. Show' that every non empty subset of a chain is a sublattice. (7.5)

4. (a) Define a distributive lattice. Show that every chain is a distributive lattice. (7.5)

(b) Let $(L_1, \leq_1)$ and $(L_2, \leq_2)$ be two ordered lattices. Define a relation $\leq$ on their Cartesian product $L = L_1 \times L_2$ by $(a_1, a_2) \leq (b_1, b_2)$ if and only if $a_1 \leq_1 b_1$ in L_1 and $a_2 \leq_2 b_2$ in L_2. Prove that $(L, \leq)$ is also a lattice. (7.5)

(c) Justify with an example that complement of an element in a non-distributive lattice need not be unique. (7.5)

5. (a) Construct circuits by using inverters, AND gates and OR gates to produce the output

$(x+y+z)\overline{x}\,\overline{y}\,\overline{z}$ (7.5)

(b) What is Disjunctive normal form and Conjunctive normal form? Find the DN form and CN form of the following Boolean function.

$f(x, y, z) = xy + xz + \bar{y}z$ (7.5)

(c) What is Karnaugh map? Use Karnaugh map diagram to find a minimal form of the function

$\bar{x}yzw + x\bar{y}z\bar{w} + \bar{x}\bar{y}z\bar{w} + xy\bar{z}\bar{w} + x\bar{y}\bar{z}\bar{w}$ (7.5)

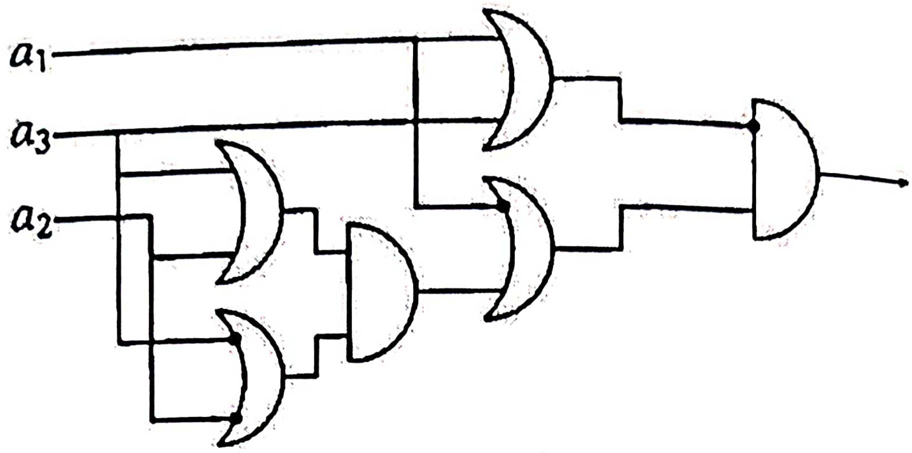
6. (a) Let $f(x,y,z) = x\bar{y}z + xyz + \bar{y}\bar{z}$. Find the implicants, prime implicants and essential prime implicants of $f(x,y,z)$ (7.5)

(b) Draw the switching circuit diagram for the following :–

(i) $p = x_1(x_2(x_3 + x_4) + x_3(x_5 + x_6))$

(ii) $p = x_1(x_2'(x_6 + x_3(x_4 + x_5')) + x_7(x_3 + x_6)x_8')$ (7.5)

(c) What is subjunction gate, NOR gate and NAND gate? Determine the Boolean polynomial of the circuit.



(7.5)