Your Roll No...............

Sr. No. of Question Paper : 4997　　　　　　E

Unique Paper Code　　　: 62351201

Name of the Paper　　　: Algebra

Name of the Course　　: **B.A. (Prog.)**

Semester　　　　　　: II

Duration : 3 Hours　　　　　Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions.

3. Attempt any **two** parts from each question.

4. Marks are indicated against each question.

1. (a) Form an equation whose roots are $-1, 2, 3 \pm 2i$.

   (6)

   (b) Solve the equation　　　　　　　　　(6)

   $$x^3 - 13x^2 + 15x + 189 = 0,$$

   being given that one of the roots exceeds another by 2.

   (c) If α, β, γ, be the roots of the equation　(6)

   $x^3 + 5x^2 - 6x + 3 = 0$, find the value of

P.T.O.

(i) $\sum \alpha^3$　　　　　　(ii) $\sum (\alpha - \beta)^2$.

2.　(a) Prove that :　　　　　　　　　　　　　　(6.5)

$2^{10} \cos^6 \theta \sin^5 \theta = \sin 11\theta + \sin 9\theta - 5\sin 7\theta - 5\sin 5\theta + 10\sin 3\theta + 10\sin\theta$.

　　(b) Sum the series :　　　　　　　　　　　　(6.5)

$\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \cdots$ to n terms, provided $\beta \neq 2k\pi$.

　　(c) State DeMoivre's theorem for rational indices and use it to solve the equation :　　　　(6.5)

$$x^7 - x^4 + x^3 - 1 = 0.$$

3.　(a) Find the characteristic roots of the matrix A where　　　　　　　　　　　　　　　　　(6)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$

　　(b) Solve the system of linear equations　　　(6)

$$2x - 5y + 7z = 6$$
$$x - 3y + 4z = 3$$
$$3x - 8y + 11z = 11$$

(c) Using Cayley Hamilton's Theorem, find the value of A^3, where   (6)

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$$

4. (a) Let X and Y be two subspace of a vector space V.   (6.5)

　　(i) Prove that the intersection $X \cap Y$ is also subspace of V.

　　(ii) Show that the union $X \cup Y$ need not be a subspace of V.

(b) Let $V = F[a, b]$ be the set of all real valued functions defined on the interval $[a, b]$. For any f and g in V, c in R, we define

$$(f + g)(x) = f(x) + g(x),$$

$$(c.f)(x) = cf(x)$$

Prove that V is a vector space over R, where R denotes the set of real numbers.   (6.5)

(c) Show that the vectors $v_1 = (1,1,1)$, $v_2 = (1,1,0)$, $v_3 = (1,0,0)$ form a spanning set of $R^3(R)$, where R denotes the set of real numbers.   (6.5)

P.T.O.

5. (a) Find the multiplicative inverse of the given elements (if it exists) if it does not exist, give the reason

    (i) [12] in Z_{16} (ii) [38] in Z_{83} (6)

    (b) Find the order of each of the following permutations

    (i) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 5 & 3 & 2 & 1 \end{pmatrix}$

    (ii) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 7 & 5 & 1 & 8 & 2 & 3 \end{pmatrix}$

    (c) Let G be a group. Prove that G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a, b $\in$ G.   (6)

6. (a) Prove that the set S = {0, 2, 4, 6, 8} is an abelian group with respect to addition modulo 10.   (6.5)

    (b) Let G be the group of all 2×2 invertible matrices with real entries under the usual matrix multiplication. Show that subset S of G defined by

    $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, b = c \right\}$, does not form a subgroup of G.   (6.5)

    (c) Show that $Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$ is a subring of R, where R is a set of real numbers & Q is set of rational numbers.   (6.5)