[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4749 E

Unique Paper Code : 32357614

Name of the Paper : DSE-3 MATHEMATICAL FINANCE

Name of the Course : B.Sc. (H) Mathematics CBCS (LOCF)

Semester : VI

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory and carry equal marks.

4. Use of Scientific calculator, Basic calculator and Normal distribution tables all are allowed.

1. (a) Explain Duration of a zero-coupon bond. A 5-year bond with a yield of 12% (continuously compounded) pays a 10% coupon at the end of each year.

(i) What is the bond's price?

(ii) Use duration to calculate the effect on the bond's price of a 0.1% decrease in its yield? (You can use the exponential values: $e^x$ = 0.8869, 0.7866, 0.6977, 0.6188, and 0.5488 for x = –0.12, –0.24, –0.36, –0.48, and –0.60, respectively)

(b) Portfolio A consists of 1-year zero coupon with a face value of ₹2000 and a 10-year zero coupon bond with face value of ₹6000. Portfolio B consists of a 5.95-year zero coupon bond with face value of ₹5000. The current yield on all bonds is 10% per annum.

(i) Show that both portfolios have the same duration.

(ii) What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?

(You can use the exponential values: $e^x$ = 0.905, 0.368, 0.552, 0.861, 0.223 and 0.409 for x = –0.1, –1.0, –0.595, –0.15, –1.5 and –0.893 respectively)

(c) Explain difference between Continuous Compounding and Monthly Compounding. What rate of interest with continuous compounding is equivalent to 15% per annum with monthly compounding?

(d) (i) "When the zero curve is upward sloping, the zero rate for a particular maturity is greater than the par yield for that maturity. When the zero curve is downward sloping the reverse is true." Explain.

(ii) Why does loan in the repo market involve very little credit risk?

2. (a) Explain Hedging. How is the risk managed when Hedging is done using?

(i) Forward Contracts; (ii) Options

(b) (i) Suppose that a March call option to buy a share for ₹50 costs ₹2.50 and is held until March. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised?

(ii) It is May, and a trader writes a September put option with a strike price of ₹20. The stock price is ₹18, and the option price is ₹2. Describe the trader's cash flows if the option is held until September and the stock price is ₹25 at that time.

(c) Write a short note on European put options. Explain the payoffs in different types of put option positions with the help of diagrams.

(d) (i) A trader writes an October put option with a strike price of ₹35. The price of the option is ₹6. Under what circumstances does the trader make a gain.

(ii) A company knows that it is due to receive a certain amount of a foreign currency in 6 months. What type of option contract is appropriate for hedging?

3. (a) Draw the diagrams illustrating the effect of changes in volatility and risk-free interest rate on both European call and put option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the vega of a European call and the vega of a European put on a non-dividend-paying stock.

(c) A European call option and put option on a stock both have a strike price of ₹20 and an expiration date in 3 months. Both sell for ₹3. The risk-free interest rate is 10% per annum, the current stock price is ₹19, and a ₹1 dividend is expected in 1 month. Identify the arbitrage opportunity open to a trader? ($e^{-0.0083} = 0.9917$)

(d) Find lower bound and upper bound for the price of a 1-month European put option on a non-dividend-paying stock when the stock price is ₹30, the strike price is ₹34, and the risk-free interest rate is 6% per annum? Justify your answer with no arbitrage arguments, ($e^{-0.005} = 0.9950$)

4. (a) Consider the standard one-period model where the stock price goes from $S_0$ to $S_0u$ or $S_0d$ with $d < 1 < u$, and consider an option which pays $f_u$ or $f_d$ in each case, and assume that the interest rate is r and time to maturity is T. Derive the formula for the no-arbitrage price of the option.

(b) A stock price is currently ₹40. It is known that at the end of one month it will be either ₹42 or ?38. The risk-free interest rate is 6% per annum with continuous compounding. Consider a portfolio consisting of one short call and Δ shares of the stock. What is the value of Δ which makes the portfolio riskless? Using no-arbitrage arguments, find the price of a one-month European call option with a strike price of ₹39? (You can use exponential value: $e^{0005} = 1.005$)

(c) Construct a two-period binomial tree for stock and European call option with

$S_0 = ₹100$, $u = 1.3$, $d = 0.8$, $r = 0.05$, $T = 1$ year, $K = ₹95$

and each period being of length $\Delta t = 0.5$ year. Find the price of the European call. If the call was American, will it be optimal to exercise the option early? Justify your answer. ($e^{-0.025} = 0.9753$)

(d) What do you mean by the volatility of a stock? How can we estimate volatility from historical prices of the stock?

5. (a) Let $S_0$ denote the current stock price, $\sigma$ the volatility of the stock, r be the risk-free interest rate and T denote a future time. In the Black-Scholes model, the stock price $S_T$ at time T in the risk-neutral world satisfies

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right]$$

where $\phi(m, v)$ denotes a norma distribution with mean m and variance v.

Using risk-neutral valuation, derive the Black-Scholes formula for the price of a European call option on the underlying stock S, strike price K and maturity T.

(b) A stock price follows log normal distribution with an expected return of 16% and a volatility of 35%. The current price is ₹38. What is the probability that a European call option on the stock with an exercise price of ₹40 and a maturity date in six months will be exercised? (You can use values: $\ln(38) = 3.638$, $\ln(40) = 3.689$)

(c) What is the price of a European call option on a non-dividend-paying stock when the stock price is ₹69, the strike price is ₹70, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?

(You can use exponential values: $e^{-0.0144} = 0.9857$, $\acute{e}^{-0.025} = 0.9753$ )

(d) A stock price is currently ₹40. Assume that the expected return from the stock is 15% and that its volatility is 25%. What is the probability distribution for the rate of return (with continuous compounding) earned over a 2-year period?

6. (a) Discuss theta of a portfolio of options and calculate the theta of a European call option on a non-dividend-paying stock where the stock price is ₹49, the strike price is ₹50, the risk-free interest rate is 5% per annum and the time to maturity is 20 weeks, and the stock price volatility is 30% per annum. ($\ln(49/50) = -0.0202$ )

(b) (i) Explain stop-loss hedging scheme.

(ii) What does it mean to assert that the delta of a call option is 0.7? How can a short position in 1,000 options be made delta neutral when the delta of each option is 0.7?

(c) Find the payoff from a butterfly spread created using call options. Also draw the profit diagram corresponding to this trading strategy.

(d) Companies X and Y have been offered the following rates per annum on a ₹5 million 10-year investment :

| | Fixed rate | Floating rate |
|---|---|---|
| Company X | 8.0% | LIBOR |
| Company Y | 8.8% | LIBOR |

Company X requires a fixed-rate investment; Company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and that will appear equally attractive to X and Y.