[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4945 H

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A./B.Sc. (Prog.) with Mathematics as Non-Major/ Minor – DSC

Semester : II

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **any two** parts from each question.

3. **All** questions carry equal marks.

4. Use of simple calculator is allowed.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $|x.y| \leq ||x|| \; ||y||$. Also verify it for the vectors $x = [2,2,1]$ and $y = [-4,0,3]$ in $\mathbb{R}^3$. (5.5+2)

(b) Solve the following system of linear equations using Gaussian Elimination method. Also indicate whether the system is consistent or inconsistent.

$$-5x - 2y + 2z = 14$$

$$3x + y - z = -8$$

$$2x + 2y - z = -3$$

(6.5+1)

(c) Find the quadratic equation $y = ax^2 + bx + c$ that goes through the points $(-2,20)$, $(1,5)$ and $(3,25)$. (7.5)

2. (a) Find the reduced row echelon form of the following matrix : (7.5)

$$\begin{bmatrix} 1 & -3 & 2 & -4 & 8 \\ 3 & -9 & 6 & -12 & 24 \\ -2 & 6 & -5 & 11 & -18 \end{bmatrix}$$

(b) Determine whether the vector $[7,1,18]$ is in the row space of the given matrix:

$$\begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}$$ (7.5)

(c) Find the characteristic polynomial, eigenvalues and corresponding eigenvectors for the given matrix:

$$\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$$ (2+2+3.5)

3. (a) Use the Diagonalization Method to determine whether the following matrix is diagonalizable.

$$\begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix}$$ (7.5)

(b) Show that the set $P_3$ of all real polynomials of degree $\leq 3$ is a vector space under the usual (term-by-term) operations of addition and scalar multiplication. (7.5)

(c) Define span of S, where S a nonempty subset of a vector space V. Determine span (S) where S = {(1,1,0), (2,1,3)} is a subset of $R^3$. Also examine whether the following vectors of $R^3$ are in span (S): (i) (0,0,0); (ii) (1,2,3).

(2+3.5+1+1)

4. (a) Define a linearly independent set of vectors in a vector space. Check whether the following subsets of $R^3$ are linearly independent or not.

(i) $\{(1,2,3), (2,3,1), (3,5,4)\}$

(ii) $\{(1,0,0), (0,0,-5)\}$ (2.5+2.5+2.5)

(b) Define a finite dimensional vector space. Let W be the solution set to the matrix equation $AX = 0$,

where $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$.

Show that (i) W is a subspace of $R^3$.

(ii) Find a basis for W. (2+3+2.5)

(c) Let $P_2$ be the vector space of all real polynomials of degree $\leq 2$. Show that

$S = \{1, x + 1, 2x + x^2\}$ is a basis of $P_2$. (7.5)

5. (a) Check if the following mappings defined on $M_{22}$ the set of $2 \times 2$ real matrices, are linear transformation or not. Prove it or give a counter example to disprove.

(i) $f : M_{22} \rightarrow R$ defined as $f(A) = \text{trace}(A)$, where trace(A) is the sum of diagonal elements of the matrix A.

(ii) $g : M_{22} \rightarrow R$ defined as $g(A) = \det(A)$.

(4+3.5)

(b) Find the matrix of linear transformation $f: P_3(x) \rightarrow R^3$ with respect to standard ordered bases defined as :

$$f(a\,x^3 + b\,x^2 + c\,x + d) = (a + 2b - c,\ 2b + d,\ a - c + d)$$

(7.5)

(c) Define a linear transformation from a vector space V to W. If $T : V \rightarrow W$ is a linear transformation and S is a subspace of W, then show that the set $T^{-1}(S) = \{v \in V \mid T(v) \in S\}$ is a subspace of V.

(2+5.5)

6. (a) Consider the linear transformation $L : M_{22} \rightarrow M_{32}$ defined as :

$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{pmatrix} a & 0 \\ 0 & 0 \\ 0 & b \end{pmatrix}$, show that dim(Range(L)) + dim(Ker(L)) = $\dim(M_{22})$. (7.5)

(b) Let $L : P(x) \rightarrow P(x)$ be a linear operator defined as $L(p(x)) = x\,p(x)$, where P(x) denotes the vector space of real polynomials. Show that L is one-one but not onto. (4+3.5)

(c) For the linear transformation $L : R^3 \rightarrow R^3$ defined as, $L(v) = A.\, v$, where

$$A = \begin{pmatrix} -9 & 2 & 1 \\ -6 & 1 & 1 \\ 5 & 0 & -2 \end{pmatrix}$$

Determine, whether L is an isomorphism or not.

(7.5)