Sl No of QP : 4905

Unique Paper Code : 2362572401
Name of the paper : DSC- Optimization Techniques
Name of the Course : B.Sc. Mathematical Sciences
Semester : IV
Duration: 3 Hours Maximum Mark: 90

Instructions for Candidates.

1. Attempt any Five questions from Q.1 to Q.7.

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1. **(a)** Consider the following unconstrained non-linear programming problem:

$f(x) = x_1^2 - x_1x_2 + 2x_2^2 - 2x_1 + e^{x_1+x_2}$.

Write the first order necessary optimality condition. Is this condition sufficient for optimality? (6)

(b) Use Newton's method to minimize $f(x_1, x_2) = 8x_1^2 - 4x_1x_2 + 5x_2^2, (x_1, x_2) \in \mathbf{R}^2$. (12)

The initial point is $x_0 = (5, 2)^t$.

2. **(a)** Find all points of local minima, local maxima and points of inflection of the function

$f(x) = (x^2 - 1)^3$

(6)

(b) Determine the definiteness of the quadratic form: (12)

$F(x) = 3x_1^2 + 6x_1x_3 + x_2^2 - 4x_2x_3 + 8x_3^2$.

3. **(a)** Check whether the following function is convex, concave or neither: (6)

$f(x_1, x_2) = (x_1 + x_2)e^{x_1+x_2}$ $x_1 > 0, x_2 > 0$.

(b) Let S be a non-empty open convex set in $\mathbf{R^n}$ and f: S → **R** be a differentiable function. Then f is convex iff $(\nabla f(x_1) - \nabla f(x_2))^t(x_1 - x_2) \geq 0$ for each $x_1, x_2 \in S$. (12)

4. **(a)** What is goal programming? Describe the weighted and pre-emptive goal programming.

(6)

(b) Harrison Electric produces two products popular with home renovators, old fashioned chandeliers and ceiling fans. Both chandeliers and fans require a two-step production process involving wiring and assembly. It takes about 2 hrs to wire a chandelier and 3 hrs to wire a fan. Final assembly of the chandelier and fan require 6 and 5 hrs respectively. The production capability is such that only 12 hrs of wiring and 30 hrs of assembly time are available. Each chandelier produced nets the firm \$7 and each fan \$6. The Harrison's management wants to achieve the following goals with the given priorities:

- P 1: Reach a profit as much above \$30 as possible.

- P 2: Fully use wiring department hours available.
- P 3: Avoid assembly department overtime.
- P 4: Produce at-least 7 ceiling fans.

Formulate and solve the above goal programming problem using graphical method. (12)

5. Solve the following goal programming problem by using Modified Simplex method: (18)

$$\text{Minimize } Z = P_1 d_1^- + P_2(8d_2^- + 6d_3^-) + P_3 d_1^+$$

Subject to:

$$2x_1 + x_2 + d_1^- - d_1^+ = 16$$
$$x_1 + d_2^- - d_2^+ = 7$$
$$x_2 + d_3^- - d_3^+ = 10$$
$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

6. **(a)** Define Integer programming problem and describe its various types. (6)

(b) A corporation is considering four possible investment opportunities. The following table presents information about the investment (in Rs. thousand) profits.

Project	Present Value of Expected Return	Capital Required Year-wise by Projects		
		Year 1	Year 2	Year 3
1	6,500	700	550	400
2	7,000	850	550	350
3	2,250	300	150	100
4	2,500	350	200	-
Capital available for investment		1,200	700	400

In addition, projects 1 and 2 are mutually exclusive. Formulate an Integer programming model to determine which projects should be accepted and which should be rejected in order to maximize the present value from the accepted projects. (12)

7. Use Branch and Bound method to solve the following integer programming problem.

$$\text{Minimize } Z = 4x_1 + 3x_2$$

Subject to:

$$5x_1 + 3x_2 \geq 30$$
$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$x_1, x_2 \geq 0 \text{ and are integers}$$

non-integer optimal solution is $x_1 = 4, x_2 = \frac{10}{3}, Z = 26.$

(18)