[This question paper contains 8 printed pages.]

Your Roll No..............

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Sr. No. of Question Paper : 5874

Unique Paper Code : 2354002004

Name of the Paper : Linear Programming

Name of the Course : GE

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) Solve graphically :

Minimize $z = 2x_1 - 3x_2$

subject to $x_1 + x_2 \leq 5$

$2x_1 + x_2 \geq 4$

$x_2 \leq 2$

$x_1, x_2 \geq 0$

(b) Find all basic feasible solution of the following system of equations :

$$2x_1 + x_2 - 2x_3 = 2$$

$$3x_1 - 2x_2 + 4x_3 = 10.$$

(c) Show that the set $S = \{(x,y) \in R^2 : x^2 + 2y^2 \leq 4\}$ is a convex set.

2. (a) Solve the following linear programming problem:

Maximize $z = 2x_1 + 5x_2 - 3x_3$

subject to
$$x_1 + 2x_2 - x_3 \leq 6$$
$$2x_1 - x_2 + x_3 \leq 12$$
$$x_1 + x_2 + x_3 \leq 4$$
$$x_1, x_2, x_3 \geq 0$$

(b) Solve the following linear programming problem :

Maximize $z = 2x_1 + 4x_2 - 3x_3$

subject to
$$2x_1 + x_2 \leq 4$$
$$3x_1 + x_2 + 4x_3 = 8$$
$$x_1, x_2, x_3 \geq 0$$

(c) Using Simplex method, solve the following linear programming problem :

Minimize $z = -x_1 + 3x_2$

subject to $x_1 + 2x_2 \geq 3$

$2x_1 - 3x_2 \geq -6$

$x_1 \leq 2$

$x_1, x_2 \geq 0.$

3. (a) Using simplex method, show that the following linear programming has no solution

Maximize $z = 4x_1 + x_2$

subject to $2x_1 + 3x_2 \geq 12$

$x_1 + x_2 \leq 3$

$3x_1 + x_2 \leq 6$

$x_1, x_2 \geq 0.$

(b) Write the dual of the following linear programming problem :

Minimize $z = x_1 + 3x_2 + 5x_3$

subject to $-2x_1 + x_2 + 3x_3 \geq 5$

$3x_1 + 2x_3 \leq 4$

$x_1 + 2x_2 + x_3 = 6$

$x_1 \leq 0$, x_2 unrestricted in sign, $x_3 \geq 0$.

(c) Show that the Dual of the Dual of MIN-problem is the MIN-problem itself.

4. (a) Find an Initial basic feasible solution of the following cost minimization Transportation problem :

(i) Using North West Comer Rule

(ii) Using Matrix Minima/Least Cost Method

	D_1	D_2	D_3	D_4	Supply
O_1	5	4	3	5	50
O_2	6	7	7	6	70
O_3	9	8	8	9	80
Demand	60	50	40	50	

(b) Find an initial basic feasible of the following cost minimization transportation problem using Vogel's Approximation Method and further find the optimal solution using MODI Algorithm.

	D_1	D_2	D_3	D_4	Supply
O_1	2	4	5	1	20
O_2	7	3	4	6	15
O_3	5	1	6	1	25
Demand	10	20	25	5	

(c) Solve the following Cost Minimisation Assignment Problem of assigning Jobs to Machines :

Jobs

Machines

	J_1	J_2	J_3	J_4	J_5
M_1	11	6	14	16	17
M_2	7	13	22	7	10
M_3	10	7	3	2	2
M_4	4	10	8	6	11
M_5	13	15	16	10	18

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5. (a) Solve the given Assignment Problem to Maximize the Sales :

	I	II	III	IV
A	50	40	60	45
B	35	50	45	40
C	40	60	50	35
D	45	45	60	70

(b) Consider a game with following pay-off matrix :

Player B

$$\textbf{Player A}\begin{bmatrix} 5 & 0 & -3 \\ 3 & 1 & 2 \\ -4 & -2 & 6 \end{bmatrix}$$

Determine the saddle points, the best strategies for each player, and the value of game.

(c) Define saddle point of a game whose pay-off matrix is $A = (a_{ij})_{m \times n}$. Find the range of x and y such that the cell (2, 3) is a saddle point of the game whose pay-off matrix is

$$\text{Player A}\begin{array}{c}\text{Player B}\\ \begin{bmatrix} 1 & 4 & -7 & 2 \\ 5 & 3 & 2 & x \\ -1 & 0 & y & 8 \end{bmatrix}\end{array}$$

6. (a) Find saddle points, if any, for the following game with pay-off matrix :

$$\text{Player I}\begin{array}{c}\text{Player II}\\ \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}\end{array}$$

Hence, or otherwise solve the game.

(b) Solve the following game using Principle of Dominance :

$$\text{Player X}\begin{array}{c}\text{Player Y}\\ \begin{bmatrix} 2 & 4 & 3 & 8 & 5 \\ 4 & 6 & 2 & 6 & 7 \\ 7 & 5 & 7 & 7 & 6 \\ 3 & 1 & 7 & 4 & 2 \end{bmatrix}\end{array}$$

(c) Transform the following game problem involving 'two-person zero sum game into its equivalent pair of linear programming problems for player A and player B

$$\text{Player A} \overset{\text{Player B}}{\begin{bmatrix} 1 & 5 & -2 \\ 4 & 1 & -3 \\ 2 & -1 & 2 \end{bmatrix}}$$