[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1608 **H**

Unique Paper Code : 12271202

Name of the Paper : Mathematical Methods for Economics-II

Name of the Course : **B.A. (Hons.) Economics**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. There are **4** questions in all.

3. **All** questions are compulsory.

4. All parts of a question must be answered together.

5. Use of simple calculator is allowed.

1. Answer any **four** of the following : (4×5=20)

(a) The marginal cost (in lakhs of rupees) of producing a motor car is given by $6 + 4x^2 + 1.5e^x$, where x is the quantity produced. Determine the total cost producing 5 motor cars if the fixed cost is Rs. 7 Lakhs. It is given that $e^x = 0.006$.

(b) (i) Suppose Y = Y(t) is national product, C(t) is consumption at time t, and $\bar{I}$ is Investment, which is constant. Suppose $\dot{Y} = \alpha(C + \bar{I} - Y)$ and C = aY + b, where a, b and a are positive constants with a < 1. Derive a differential equation for Y.

(ii) Find its solution when $Y(0) = Y_0$ is given. What happens to Y(t) as $t \to \infty$?

(c) Let Y_t denote national income, I_t total investment, and S_t total saving – all in period t. Suppose savings are proportional to national income, and that investment is proportional to the change in income from period t to t+1. Then, for t = 0,1,2 ......,

(i) $S_t = \beta Y_t$

(ii) $I_{t+1} = \alpha(Y_{t+1} - Y_t)$

(iii) $S_t = I_t \quad \alpha, \beta > 0 \quad \beta < \alpha$

Deduce a difference equation determining the path of Y_t, given Y_0 and solve it.

(d) Find the general form of a function f whose second derivative $f''(x)$ is equal to x^2. If we require in addition that $f(0) = -1$, what is $f(x)$?

(e) Find the area of the region bounded by $y = x^2$ and $y = x$.

2. Answer any **four** of the following : (4×5=20)

(a) Show that the production function $Y = Ax^{\alpha}y^{\beta}$ where $A, \alpha, \beta > 0$ has elasticity of substitution 1 everywhere.

(b) A firm sells 2 brands X and Y of a soap. The outputs and prices are denoted by x, y, p and q respectively. The demand functions of the two brands are : $x = 100 - 2p + 5q$ and $y = 80 + 4p - 3q$. Suppose brand X sells for rupees 15 per unit and Y for 12 per unit. Estimate the approximate change in revenue if the prices are increased by 1 per unit for X and 1.5 per unit for Y and compare it with the actual change in revenue.

(c) Draw the level curves for the function

$$f(x,y) = \frac{2x - 2y}{x^2 + y^2 + 1} \text{ for levels } k = 0, 1.$$

[For PWD candidates in lieu of part c]

Find the equation of the tangent plane at a point (1, 1, 5) to the function $x^2 + 2xy + 2y^2$

(d) Examine the definiteness of the following quadratic forms :

(i) $f(x, y) = x^2 + 2xy + y^2$

(ii) $-x^2 + y^2$ subject to $2x - 3y = 5$

(c) Find and sketch the domain of the following functions :

(i) $f(x,y)=\sqrt{x^2-y}$

(ii) $f(x,y)=\sqrt{y+4}-\sqrt{x+1}$

[For PWD candidates in lieu of part c]

Find the domain of the following functions :

(i) $f(x,y)=\sqrt{x^2-y}$

(ii) $f(x,y)=\sqrt{y+4}-\sqrt{x+1}$

(iii) $f(x, y) = \ln(y^2 - x)$

Answer any **four** of the following : (4×7=28)

(a) Based on the table, classify each of the following points as a local maximum, local minimum, saddle point, not a critical point, or not enough information to classify.

Point	f'_x	f'_y	f''_{xx}	f''_{xy}	f''_{yy}
A	1	2	3	4	5
B	0	0	0	0	0
C	0	0	1	2	3
D	0	0	0	1	0
E	0	0	0	-1	0

(b) Find the extreme points of the function f(x,y) 9x + 8y – 6(x + y)² subject to 0 ≤ x ≤ 5, 0 ≤ y and x – 2y ≥ –2.

(c) Find all extreme points and/or saddle points of t function :

$$3x^2y + y^3 - 3x^2 - 3y^2 + 2$$

(d) Define quasi-concave and quasi-convex function Prove geometrically or otherwise that $y = x^2$: x ≥ is both quasi-concave and quasi-convex.

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(c) Find the maximum and minimum values of $f(x, y) = x + y$ subject to $x^2 + y^2 = 4$ using Lagrange multiplier method. Verify the second order conditions.

Answer any **one** of the following : (1×7=7)

(a) The Eco club Econ manufactures eco-friendly inexpensive notebooks and diaries. Each notebook takes 4 Kgs of paper pulp and 2 litres of water solution in its preparation while each diary requires 3 Kgs of paper pulp and 1 litre of water solution. The club has 240 Kgs of paper pulp and 100 litres of water solution available with it. Each notebook sold gives a profit of Rs. 7 and each diary gives a profit of Rs. 5. Using linear programming, find the optimal combination of notebooks and diaries to be produced in order to maximize the profit.

(b) Is there a solution to the following linear programming problem?

Maximize $x_1 + x_2$ subject to

$$\begin{cases} -x_1 + x_2 \leq -1 \\ -x_1 + 3x_2 \leq 3 \end{cases} \quad x_1 \geq 0 \quad x_2 \geq 0$$

Will a solution exist if the objective function is changed to $z = -x_1 - x_2$?

Solve wherever possible.