[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper: 4991

E

Unique Paper Code

: 62367602

Name of the Paper

: Integer Programming and

Theory of Games

Name of the Course

: B.A. (Prog.)

Semester

VI

Duration: 3 Hours

Maximum Marks: 75

## Instructions for Candidates

- 1. Write your Roll No. on the top immediately on receipt of this question paper.
- 2. Attempt any five questions in all.
- 3. All questions carry equal marks,
- 1. (a) (i) Explain Integer Programming problem (IPP).

  Discuss two types of IPP With an example and state how it is an special case of Linear Programming problem

(ii) What do you understand by 0-1 programming problem. Discuss Capital budgeting and its importance? (5+5)

(b) Define the Saddle point. The pay-off matrix of a game is given below. Find the solution of the game of A and B. (5)

|     | I  | II | 111 | IV |
|-----|----|----|-----|----|
| I   | 20 | 15 | 12  | 35 |
| II  | 25 | 14 | 8   | 10 |
| III | 40 | 2  | 10  | 5  |
| IV  | -5 | 4  | 11  | 0  |

2. (a) (i) Define Gomory's Constraint.

(ii) Determine the range of value of p and q that will make the payoff element  $a_{22}$  a saddle point for the game whose payoff matrix  $(a_{ij})$  is given below: (2+5)

| 2  | 4 | 5 |
|----|---|---|
| 10 | 7 | q |
| 4  | р | 8 |

(b) Find the optimum integer solution to the following LPP. (8)

Max 
$$Z = X_1 + X_2$$
  
Subject to  $3X_1 + 2X_2 \le 5$   
 $X_2 \le 2$   
 $X_1, X_2 \ge 0$  and are integers.

- 3. (a) Write any three real-world situations where Integer programming problem (0-1 programming) can be used. (7)
  - (b) A corporation is considering the four possible investment opportunities. The following table presents information about the investment (in Rs. Thousand) profits:

| Project                          | Present value of expected return | Year 1 | Year 2 | Year 3 |
|----------------------------------|----------------------------------|--------|--------|--------|
| 1                                | 650                              | 700    | 550    | 400    |
| 2                                | 700                              | 850    | 550    | 350    |
| 3                                | 225                              | 300    | 150    |        |
| 4                                | 250                              | 350    | 200    | 100    |
|                                  |                                  | 1200   |        | •      |
| Capital available for investment |                                  |        | 700    | 400    |

In addition, projects 1 an 2 are mutually exclusive and project 4 I contingent on the prior acceptance of project 3. Formulate an Integer programming model to determine which projects should be accepted and which projects should be rejected to maximize present value form accepted project.

- 4. (a) Explain the concept of Branch and bound method. (7)
  - (b) Use branch and bound method to solve the following LPP: (8)

Maximize  $Z = 7X_1 + 9X_2$ 

subjects to:  $-X_1 + 3X_2 \le 6$ 

 $7X_1 + X_2 \le 35$ 

 $X_2 \leq 7$ 

 $X_1$ ,  $X_2 \ge 0$  and all are integers.

- (a) Explain the decision under uncertainty and decision under risk. Discuss different methods under each category.
  - (b) A businessman has three alternatives open to him each of which can be followed by any of the four possible events. The conditional payoffs (in Rs.) for each action-event combination are given below:

| Alternative | Payoffs conditional on events |    |     |    |
|-------------|-------------------------------|----|-----|----|
|             |                               |    |     |    |
|             | A                             | В  | С   | D  |
| X           | 8                             | 0  | -10 | 6  |
| Y           | -4                            | 12 | 18  | -2 |
| Z           | 14                            | 6  | 0   | 8  |

Determine which alternative should be businessman choose, if he adopts the maximin criterion. (7)

6. (a) Solve the following integer programming problems using Gomory's cutting plane algorithm or by branch or bund method. (8)

Maximize 
$$Z = 2X_1 + 3X_2$$

subjects to: 
$$X_1 + 3X_2 \le 9$$
 
$$3X_1 + X_2 \le 7$$
 
$$X_1 - X_2 \le 1$$

 $X_1, X_2 \ge 0$  and all are integers

(b) Reduce the following game 2×5 to 252 by dominance and ten solve by graphical method
(7)

|    | I  | II | III | IV | V |
|----|----|----|-----|----|---|
| I  | 2  | -1 | 5   | -2 | 6 |
| II | -2 | 4  | -3  | 1  | 0 |

7. (a) A salesman has to visit five cities A, B, C, D and
E. The distances (in hundred km) between the
five cities are as follows:

|   | A  | В  | C               | D  | E        |
|---|----|----|-----------------|----|----------|
| A | 00 | 17 | 16              | 18 | 14       |
| В | 17 | 00 | 18              | 15 | 16       |
| С | 16 | 18 | $\infty$        | 19 | 17       |
| D | 18 | 15 | 19              | ∞  | 18       |
| E | 14 | 16 | 17              | 18 | 00       |
|   |    | L  | (S.11-52-29/28) | 1  | <u> </u> |

If the salesman starts from city A and has to come back to city A, which route should he select so that total distance travelled by him is minimized? (7)

(b) Deduce the following game using the concept of dominance principle and then solve it. (8)

|     | I | 11 | III | IV ' |
|-----|---|----|-----|------|
| I   | 3 | 2  | 4   | 0    |
| II  | 3 | 4  | 2   | 4    |
| III | 4 | 2  | 4   | 0    |
| IV  | 0 | 4  | 0   | 8    |