Jan 2024

[This question paper contains 4 printed pages.]

Your Roll No..............

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Sr. No. of Question Paper : 896

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **Two** parts from each question.

3. **All** questions carry equal marks.

1. (a) If $f(x) = |x - 1|$, show that f is continuous but not differentiable at $x = 1$.

   (b) Find the n^{th} derivative of $y = \cos x \cos 2x \cos 3x$.

P.T.O.

(c) If $u = \frac{x^2y^2}{x^2+y^2}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$ and hence prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u$$

2. (a) Let $f(x) = \begin{cases} 1 - x & , x < 1 \\ x^2 - 1 & , x \geq 1 \end{cases}$

show that f is continuous but not differentiable at x = 1.

(b) If y = $\sin^{-1} x$, prove that

$$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0$$

(c) If $u = \tan^{-1} \frac{xy}{\sqrt{1+x^2+y^2}}$, prove that

$$\frac{\partial^2 u}{\partial x\,\partial y} = \frac{1}{(1+x^2+y^2)^{\frac{3}{2}}}.$$

3. (a) State Taylor's theorem with Cauchy's form of remainder. Prove that

$$\log(1 + x) = x - \frac{x^2}{2} + \cdots + \frac{(-1)^{n-2}x^{n-1}}{(n-1)} + \frac{(-1)^{n-1}x^n}{n(1+\theta x)^n}$$

(b) Discuss applicability of Rolle's theorem for the following functions:

(i) $f(x) = x^2, x \in [-1, 1]$.

(ii) $f(x) = |x|, x \in [-2, 2]$.

(c) Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$.

4. (a) State Lagrange's Mean Value Theorem. Verify it for

$$f(x) = x^3 - 5x^2 - 3x, x \in [1,3].$$

(b) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$.

(c) Find the Taylor Series expansion for $f(x) = \sin(x)$.

5. (a) Find all the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0.$$

(b) Trace the curve

$$x^3 + y^3 = a^2x, a > 0.$$

(c) If $u_n = \int_0^{\frac{\pi}{2}} \cos^n x\, dx$, show that $u_n = \frac{n-1}{n} u_{n-2}$.

Hence evaluate u_6.

6. (a) Prove that the curve

$$(x-a)^2(x-b) = y^2, \; a>0, \; b>0$$

has at $x = a$, a node if $a > b$, a cusp if $a = b$ and a conjugate point if $a < b$.

(b) If $u_{n,m} = \int_0^{\frac{\pi}{2}} \sin^n x \cos^m x dx$, show that

$$u_{n,m} = \frac{n-1}{m+n} u_{n-2,m}.$$

Also evaluate $u_{3,4}$.

(c) Trace the curve

$$x^2(x^2 - 4a^2) = y^2(x^2 - a^2), \; a > 0.$$