[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 916 **G**

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Two** parts from each question.
3. **All** questions carry equal marks.

1. (a) If $f(x) = \begin{cases} \frac{x}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

show that f is continuous but not differentiable at x = 0.

P.T.O.

(b) If $y = e^{\tan^{-1}x}$, prove that

$$(1+x^2)y_{n+2} + +\{2(n+1)x-1\}y_{n+1} + n(n+1)y_n = 0$$

(c) State Euler's theorem and verify it for $z = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$.

2. (a) If $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, check continuity of the function f at x = 0 and specify the type of discontinuity, if any.

(b) Find the n^{th} derivative of $y = \cos^2 x \sin^3 x$.

(c) If $u = \log\frac{x^4+y^4}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

3. (a) State Lagrange's mean value theorem and use it to show that

$$1 + x < e^x < 1 + xe^x, \quad x > 0.$$

(b) Prove

$$\sin ax = ax - \frac{a^3x^3}{3!} + \cdots + \frac{a^{n-1}x^{n-1}}{(n-1)!} + \frac{a^n x^n}{n!}\sin\left(a\theta x + \frac{n\pi}{2}\right),$$

$$\sin\left(\frac{(n-1)\pi}{2}\right)$$

(c) Find a, b, c so that $\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$.

4. (a) Verify Rolle's theorem for

(i) $x^3 - 6x^2 + 11x - 6,\ x \in [1,3]$

(ii) $\sin x,\ x \in [0,\pi]$.

(b) State Taylor's theorem with Lagrange's form of remainder. Find the Taylor series expansion of $f(x) = \sin x$.

(c) Evaluate $\lim_{x\to 0} \frac{\tan x - x}{x^2 \tan x}$.

5. (a) Find all the asymptotes of the curve

$$x^2y^2(x^2 - y^2)^2 = (x^2 + y^2)^3.$$

P.T.O.

(b) Trace the curve

$$y^2(a + x) = x^2(a - x), \quad a > 0.$$

(c) Find a reduction formula for

$$\int \cos^n x dx .$$

Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x dx$.

6. (a) Determine the position and nature of double points on the curve

$$x^3 - y^2 + 2x^2 + 2xy + 5x - 2y = 0.$$

(b) Obtain a reduction formula for $\int \sin^m x \cos^n x dx$.

Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x dx$.

(c) Trace the curve

$$x^2 (a - x) = ay^2, \quad a > 0.$$