[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3924 G

Unique Paper Code : 32355301

Name of the Paper : GE – III Differential Equations

Name of the Course : Generic Elective / Other than B.Sc. (H) Mathematics

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the six questions are compulsory.
3. Attempt any two parts from each question.

1. (a) Solve the following differential equations :

$$(4x + 3y + 1)dx + (x + y + 1)dy = 0, \; y(3) = -4.$$

(6.5)

(b) Solve the Initial Value problem :

$$x^2 \frac{dy}{dx} + xy = \frac{y^3}{x}, \quad y(1) = 1$$ (6.5)

P.T.O.

(c) Find the orthogonal trajectories of the family of ellipse having centre at the origin , a focus at the point (c,0) and semimajor axis of length 2c. (6.5)

2. (a) (i) Find the Wronskian of the set $\{1-x,\ 1+x,\ 1-3x\}$ and hence find their linear independence or dependence on $(-\infty,\infty)$. (6)

(ii) Solve the differential equation :

$$\frac{dy}{dx}=\frac{2+ye^{xy}}{2y-xe^{xy}}.$$

(b) Given that $y=x$ is a solution of the differential equation

$$\left(x^2-x+1\right)\frac{d^2y}{dx^2}-\left(x^2+x\right)\frac{dy}{dx}+(x+1)y=0,$$

find a linearly independent solution by reducing the order. Write the general solution also. (6)

(c) Solve the following differential equations :

(i) $x\frac{dy}{dx}-2y=2x^4,\ y(2)=8$

(ii) $(2xy^2+y)dx+(2y^3-x)dy=0.$ (6)

3. (a) Solve the Initial value Problem

$$\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}-10\frac{dy}{dx}=0,\qquad y(0)=7,\qquad \frac{dy}{dx}(0)=0,$$

$$\frac{d^2y}{dx^2}(0)=70.$$ (6.5)

(b) Find the general solution of the differential equation using method of Undetermined Coefficients

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 8\sin(3x).$$ (6.5)

(c) Find the general solution of the differential equation using method of Variation of Parameters

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec}(x).$$ (6.5)

4. (a) Given that sin x is a solution of the differential equation

$$\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$$

find the general solution. (6)

(b) Find the general solution of the differential equation by assuming $x > 0$,

$$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 0$$ (6)

(c) Find the general solution of the given linear system

$$2\frac{dx}{dt} + \frac{dy}{dt} - x - y = e^{-t}, \quad \frac{dx}{dt} + \frac{dy}{dt} + 2x + y = e^{t}.$$ (6)

5. (a) Find the solution of the linear partial differential equation

$(y-u)u_x + (u-x)u_y = x-y$, with Cauchy data $u = 0$ on $xy = 1$. (6)

(b) Find the solution of the following partial differential equation by the method of separation of variables

$u_x + u = u_y, \quad u(x, 0) = 4\exp(-3y).$ (6)

(c) Reduce the equation to canonical form and obtain the general solution

$$u_x + 2xy\, u_y = x.$$ (6)

6. (a) Find the general solution of the linear partial differential equation

$yz\, u_x - xz\, u_y + xy(x^2 + y^2)\, u_z = 0.$ (6.5)

(b) Reduce the equation

$u_{xx} - \frac{1}{c^2} u_{yy} = 0, \ c \neq 0$ where c is a constant,

into canonical form and hence find the general solution. (6.5)

(c) Reduce the following partial differential equation with constant coefficients,

$$3\, u_{xx} + 10\, u_{xy} + 3\, u_{yy} = 0$$

into canonical form and hence find the general solution. (6.5)

(1000)