[This question paper contains 8 printed pages.]

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Your Roll No...............

Sr. No. of Question Paper : 1441 F

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A. (Prog.)

Semester : II – DSC

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** question by selecting **two** parts from each question.

3. **All** questions carry equal marks.

4. Use of Calculator not allowed.

1. (a) If x and y are vectors in R^n, then prove that $||x + y|| \leq ||x|| + ||y||$.

Also verify it for the vectors x = [–1, 4, 2, 0, –3] and y = [2, 1, –4, –1, 0] in R^5. (5.5+2)

(b) Prove that for vectors x and y in R^n,

(i) $x \cdot y = \frac{1}{4}(||x + y||^2 - ||x - y||^2)$

(ii) If $(x + y) \cdot (x - y) = 0$, then $||x|| = ||y||$.

(4+3.5)

(c) Solve the systems $AX = B_1$ and $AX = B_2$ simultaneously, where

$$A = \begin{bmatrix} 9 & 2 & 2 \\ 3 & 2 & 4 \\ 27 & 12 & 22 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -6 \\ 0 \\ 12 \end{bmatrix}, \text{ and } \quad B_2 = \begin{bmatrix} -12 \\ -3 \\ 8 \end{bmatrix}$$

(7.5)

2. (a) Find the reduced row echelon form of the following matrix :

$$A = \begin{bmatrix} 2 & -5 & -20 \\ 0 & 2 & 7 \\ 1 & -5 & -19 \end{bmatrix}$$ (7.5)

(b) Express the vector $x = [2, -1, 4]$ as a linear combination of vectors $v_1 = [3, 6, 2]$ and $v_2 = [2, 10, -4]$, if possible. (7.5)

(c) Define the rank of a matrix and determine it for the following matrix :

$$B = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{bmatrix}$$ (1.5+6)

3. (a) Check if the following matrix is diagonalizable or not :

$$\begin{bmatrix} 3 & 4 & 12 \\ 4 & -12 & 3 \\ 12 & 3 & -4 \end{bmatrix}$$ (7.5)

(b) Show that the set of all polynomials P(x) forms a vector space under usual polynomial addition and scalar multiplication. (7.5)

(c) Give an example of a finite dimensional vector space. Check if the following are a vector space or not :

(i) R^2 with the addition $[x, y] \oplus [w, z] = [x + w + 1, y + z - 1]$ and scalar multiplication $a \otimes [x, y] = [ax + a - 1, ay - 2]$.

(ii) set of all real valued functions $f: R \to R$ such that $f\left(\frac{1}{2}\right)=1$, under usual function addition and scalar multiplication.

(1.5+3+3)

4. (a) Definesubspace of a vector space. Further show that intersection of two subspaces of a vector space V is a subspace of V. (1.5+6)

(b) Define a linearly independent set. Check if $S = \{(1, -1, 0, 2), (0, -2, 1, 0), (2, 0, -1, 1))$ is linearly independent set in R^4 or not.

(1.5+6)

(c) Define an infinite dimensional and finite dimensional vector space.

Consider the set of all real polynomials denoted by P(x), and the set of all real polynomials of degree at most n denoted by $P_n(x)$. Describe a basis of P(x) and $P_n(x)$ and mention if these are finite dimensional or infinite dimensional.

(2+4+1.5)

5. (a) Show that the mapping $L : M_{nn} \to M_{nn}$, defined as $L(A) = A + A^T$ is a linear operator, where M_{nn} is set of $n \times n$ matrices and A^T denotes the transpose of the matrix A. Find the Kernel of L.

(3+4.5)

(b) Let $L: R^2 \to R^3$ be a linear transformation defined as L{[a, b]) = [a – b, a, 2a + b]. Find the matrix of linear transformation A_{BC} of L, with respect to the basis B = {[1,2], [1,0]} and C = {[1,1,0], [0,1,1], [1,0,1]}. (7.5)

(c) Let L: V → W, be a linear transformation, then define Ker(L), Range(L). Further show that Ker(L) is a subspace of V and Range(L) is a subspace of W. (1.5+1.5+2.5+2)

6. (a) For the linear transformation L: $R^3 \rightarrow R^3$ defined as

$$L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Find Ker(L) and Range(L). (4+3.5)

(b) Let L: V → W be a one-to-one linear transformation. Show that if T is a linearly independent subset of V, then L(T) is a linearly independent subset of W. (7.5)

(c) For the linear transformation $L: R^2 \rightarrow R^2$, defined as :

$$L\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

Find L^{-1}, if it exists. (7.5)