[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1174 **B**

Unique Paper Code : 32355202

Name of the Paper : GE-2 Linear Algebra

Name of the Course : **Generic Elective**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Do any **two** parts from each question.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $|x \cdot y| \leq \|x\| \, \|y\|$.

   Also, verify it for the vectors x = [1, –2, 0, 2, 3], y = [0, –3, 2, –1, –1]. (6)

   (b) Let x and y be non-zero vectors in $\mathbb{R}^n$, then prove that $x \cdot y = \|x\| \, \|y\|$ if and only if y is a positive scalar multiple of x. (6)

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(c) (i) For the vectors a = [6, −5, −2] and b = [−4, -3, 2], find $\text{proj}_a b$ and verify that $b - \text{proj}_a b$ is orthogonal to a. (3)

(ii) Prove that if $(x + y) \cdot (x - y) = 0$, then $\|x\| = \|y\|$, where x and y are vectors in $\mathbb{R}^n$. (3)

(d) Use the Gauss-Jordan Method to solve the following system of linear equations

$$x + 2y + z = 8$$
$$2x + 3y + 2z = 14$$
$$3x + 2y + 2z = 13.$$

(6)

2. (a) Determine whether the vector [7,1,18] is in the row space of the matrix

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}.$$

(6.5)

(b) Find the characteristic polynomial and eigen values of the matrix

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 6 & 2 & -6 \\ 4 & 0 & -2 \end{bmatrix}$$

Is A diagonalizable? Justify. (6.5)

(c) (i) Show that the set of vectors of the form $[2a - 3b, a - 5c, a, 4c - b, c]$ in $\mathbb{R}^5$ forms a subspace of $\mathbb{R}^5$ under the usual operations. (3)

(ii) Find the eigenspace E_λ corresponding to the eigen value $\lambda = 3$ for the matrix

$$A = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix} \qquad (3.5)$$

(d) Let V be the set $\mathbb{R}^2$ with the operations addition and scalar multiplication for x, y, w, z and a in $\mathbb{R}$ defined by :

$[x, y] \oplus [w, z] = [x + w + 1, y + z - 2]$ and

$a \odot [x, y] = [ax + a - 1, ay - 2a + 2]$.

Prove that V is a vector space over $\mathbb{R}$. Find the zero vector in V and the additive inverse of each vector in V. (6.5)

3. (a) Using rank criterion, check whether the following system is consistent or not?

$$\begin{bmatrix} 1 & -2 & -3 & 4 \\ 4 & -1 & -5 & 6 \\ 2 & 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

If consistent, solve the system. (6)

(b) Use Simplified Span Method to find a simplified general form for all the vectors in span(S), where $S = \{[1, -1, 1], [2, -3, 3], [0, 1, -1]\}$ is a subset of $\mathbb{R}^3$. (6)

(c) Let $B = \{[1, -2, 1], [5, -3, 0]\}$ and

$S = \{[1, -2, 1], [3, 1, -2], [5, -3, 0], [5, 4, -5], [0, 0, 0]\}$ be subsets of $\mathbb{R}^3$.

(i) Show that B is a maximal independent subset of S.

(ii) Calculate dim(span(S)).

(iii) Does span(S) = $\mathbb{R}^3$? Justify. (6)

(d) Use the Independence Test Method to show that the subset

$S = \{x^2 + x + 1, x^2 - 1, x^2 + 1\}$ of P_2 is linearly independent. (6)

4. (a) (i) Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + ky \\ y \end{bmatrix}$$

Show that f is a linear operator.

(ii) Let V be a vector space, and let $x \neq 0$ be a fixed vector in V. Prove that the translation function $f: V \to V$ given by $f(v) = v + x$ is not linear. (6.5)

(b) Let $S = \{[1,2], [0,1]\}$ and $T = \{[1,1], [2,3]\}$ be bases for $\mathbb{R}^2$. Let $v = [1,5]$.

(i) Find the coordinate vector of v with respect to the basis T.

(ii) What is the transition matrix $P_{S \leftarrow T}$ from the basis T to the basis S.

(iii) Find the coordinate vectors of v with respect to S using $P_{S \leftarrow T}$. (6.5)

(c) Suppose $L: \mathbb{R}^3 \to \mathbb{R}^3$ is a linear operator and $L([1,0,0]) = [-3,2,4]$, $L([0,1,0]) = [5,-1,3]$ and $L[(0,0,1)] = [-4,0,-2]$. Find $L([6,2,-7])$. Find $L([x, y, z])$, for any $[x, y, z] \in \mathbb{R}^3$. (6.5)

(d) Let $L: \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 & 3 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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be a linear transformation and B = {[1,−3,2], [−4,13,−3], [2,−3,20]} and C = {[−2,−1], [5,3]} be bases for $\mathbb{R}^3$ and $\mathbb{R}^2$, respectively. Find the matrix A_{BC} of L w.r.t. B and C. (6.5)

5. (a) Consider the linear transformation L: $\mathbb{R}^3 \to \mathbb{R}^2$ given by

$L([x, y, z]) = [x + y,\ y + z]$. Show that L is onto but not one to one. (6)

(b) Find the minimum distance from the point P(1,4,−2) to the subspace W = span $\{[x,y,z]: -2x + 5y - z = 0\}$ of $\mathbb{R}^3$. (6)

(c) Find a least squares solution for the linear system AX = B, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \\ 4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ 15 \\ 14 \end{bmatrix}$$ (6)

(d) Consider a polygon associated with 2 × 5 matrix

$$\begin{bmatrix} 8 & 8 & 6 & 8 & 10 & 10 \\ 6 & 8 & 10 & 12 & 10 & 6 \end{bmatrix}.$$ Use ordinary coordinates

in $\mathbb{R}^2$ to find the new vertices after performing each indicated operation :

(i) translation along the vector [12,6].

(ii) rotation about the origin through $\theta = 90°$.

(iii) reflection about the line $y = -3x$.

(iv) scaling about the origin with scale factors of 1/2 in the x-direction and 4 in the y-direction. (6)

6. (a) Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the Linear Operator given by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -4 & 2 & 6 \\ 1 & 1 & -3 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the basis for Ker(L) and basis for Range(L). Verify Dimension Theorem. (6.5)

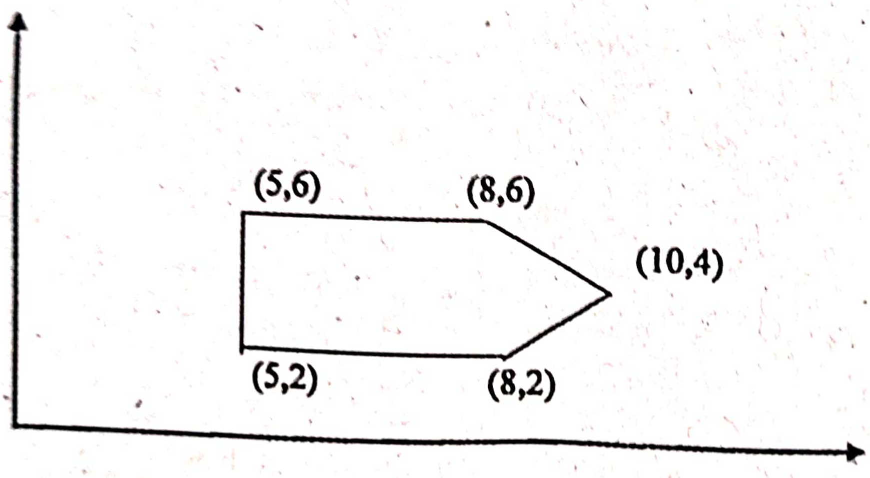
(b) For the subspace $W = \{[x, y, z]: -x + 4y - 2z = 0\}$ of $\mathbb{R}^3$, find $W^{\perp}$, the orthogonal complement of W. Verify $\dim(W) + \dim(W^{\perp}) = \dim(\mathbb{R}^3)$. (6.5)

(c) Verify that the given ordered basis B is orthonormal. Hence, for the given v, find $[v]_B$

where $v = [-2,3]$, $B = \left\{\left[\frac{-\sqrt{3}}{2}, \frac{1}{2}\right]\left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]\right\}$.

(6.5)

(d) For the given graphic, use homogeneous coordinates to find the new vertices after performing a scaling about (2,2) with scale factor of 2 in x-direction and 3 in y-direction. Sketch the final figure resulting from the movement.



(6.5)